IDENTIFICATION OF MAGNETOSTRICTION VIBRATIONS IN POWER TRANSFORMERS
(Ph. d. thesis)

Summary: The paper presents a methodology of identification and calculating vibrations of power transformers caused by magnetostriction. The method uses two different Finite Element Method numerical models for magnetostatic and mechanical calculations. The magnetostatic calculations were carried out on a 2D model with equivalent characteristics of electrotechnical steel in the overlap zone. Instantaneous magnetostriction forces acting on different parts of the transformer core were then calculated with the use of a symmetrical stress tensor. The calculated forces were transferred to a 3D orthotropic mechanical model and the resulting modes of deflections were determined. The mechanical model took into account the lamination of the transformer core. Experimental measurements of Operational Deflection Shapes were carried out parallel and the results compared. The presented method makes it possible to estimate the amplitudes and modes of vibrations caused by the magnetostriction phenomena and determine the risk of resonance amplification of vibrations as well - in the case of various technological inaccuracies of the core assembly.

1. INTRODUCTION

The growth of electric energy consumption is one of the distinct factors in present world. Nowadays, 3-phase transformers are a frequent component of the human environment and their interaction with the surroundings becomes more and more important. One of these interactions is vibrations and the resulting noise emission. The nature of magnetostriction forces causing the major part of mechanical vibrations have not yet been fully explained since the problem covers a very wide and interdisciplinary domain of applied science – electrodynamics and structural mechanics of solids.

Investigations were carried out on a power transformer, which means a 3-phase transformer connected to a standard power supply system. The model was a three-leg transformer, but the calculation methodology can be easily extended on four and five leg structures. Calculations were made for a no-load state, since then the magnetostriction noise is dominant. The method of calculating magnetic (Lorentz) forces is known and the resulting vibrations of the windings can be easily determined given a proper mechanical model.

The most important element in investigating the magnetostriction vibrations is developing the mathematical model of magnetostriction forces origin. The stress tensor description presented by Witczak [5] makes it possible to calculate the magnetostriction stress field from a known 2D magnetic field distribution and take into account the measured material anisotropic characteristics in the symmetrical stress tensor. The method was used by the author in several publications [1] [2] [6] presenting partial results of the theoretical analysis of transformer vibrations.
In the mechanical calculations the most important factor is to determine the orthotropic parameters of the laminated core and the windings. Calculating the forced vibrations is an already recognized problem in mechanics of continuous medium, knowing the time-space distribution of the input forces and equivalent characteristics of the composite structure of the core and windings.

Taking into account the complicated geometric and material construction of the investigated model the Finite Element Method was chosen, both in the domain of electromagnetics and mechanics. The main emphasis of the work was put to the exchange of calculated forces data between the two different areas of computations.

Apart from the theoretical analysis, experimental measurements of Operational Deflection Shapes were carried out for over hundred degrees of freedom, in order to confront the results.

The hypothesis to prove were:
1. Determining the magnetostriction vibrations of a power transformer requires stress tensor description of the magnetostriction forcers and the use of an orthotropic mechanical model.
2. Minor technological inaccuracies of the core assembly may cause high levels of vibrations.

Simplification assumptions:
- The transformer is supplied with a 3-phase, symmetrical, monoharmonic voltage system.
- The magnetic characteristics of electrotechnical steel are nonlinear and single valued.
- A 2D magnetostatic model with equivalent characteristics of electrotechnical steel in the overlap zone is sufficient to describe the magnetic field distribution.
- The parameters of the construction materials of the core and windings are linear and orthotropic.
- The calculations of dynamic deflections of the model are carried out with the assumption that the linearised stress tensor is symmetrical.

2. ELECTROMAGNETIC CALCULATIONS

Modern magnetic field computational programs, based on Finite Element Method, do not enable complex and sufficiently accurate calculations of big and complicated 3D structures such as the transformer core. This is mainly caused by the disproportion between the main dimensions and the thickness of electrotechnical sheets and the size of air gaps in the overlap zone. Good results could be obtained by superposition of two 2D models taking into account the influence of the overlap on the distribution of the magnetic field in the core [3].

The basis of taking into consideration the influence of the overlap is calculating a 2D model of the cross-section of the overlap region, perpendicular to the sheet surface. The second model includes the whole transformer core, in the plane parallel to the sheets, with equivalent magnetic characteristics of the material in the overlap zone, calculated from the previous model. The equivalent characteristics are applied to a much wider region than the actual size of the overlap – this enables to form bigger elements in the numerical model (Fig.1).

![Fig. 1. Geometry and boundary conditions of the 2D numerical FEM model of the core.](image-url)
The magnetostatic calculations are made step by step (time stepping) with the assumption of
the sinusoidal flux density input, adequate to a given averaged value of the maximal induction
in the legs of the core. The relations between the fluxes and the magnetic potential, which
forms the boundary conditions for FEM calculations (Fig. 1), are presented in equation (1).

\[
\begin{align*}
\frac{1}{t_k} \Phi_y(t) &= -A_y(t) \\
\frac{1}{t_k} \Phi_x(t) &= +A_y(t) - A_z(t) \\
\frac{1}{t_k} \Phi_z(t) &= +A_z(t)
\end{align*}
\] (1)

The results of magnetic field distribution calculations were accurate, compared to results
obtained by other investigators [3], who took into account the actual anisotropic characteristic
of the electrotechnical steel. Examples of the magnetic field distribution are presented in Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{magnetic_field_distribution.png}
\caption{Examples of magnetic field distribution for different time steps: a) \(\omega \tau = 0\) deg, b) \(\omega \tau = 90\) deg.}
\end{figure}

3. CALCULATIONS OF MAGNETOSTRICTION FORCES

The classical theory of elasticity for orthotropic bodies was used in order to associate
the magnetostriction mechanical stress field with the magnetic induction field [4], with the
particular application to electrotechnical sheets. Taking into account that the crystalline Goss
structure of the grain oriented electrotechnical steel is similar to a cubic symmetry, using the
Voigt notation and incorporating normalized modes of deflections it is possible to derive the
formula to calculate the elongations dependent on the value and direction of the magnetic
induction (2).

\[
\lambda(\alpha, \beta, B) = \frac{1}{2} \left( 3\beta^2 \alpha^2 - \beta^2 - \alpha^2 + \frac{1}{3} \lambda_e B^2 + \right.
\]
\[
+ \frac{1}{2} (1 - \beta^2)(1 - \alpha^2)(\lambda_{14} B^2 + \lambda_{15} B^4) +
\]
\[
+ 2\beta \alpha \sqrt{(1 - \beta^2)(1 - \alpha^2)} (\lambda_{22} B^2 + \lambda_{24} B^4)
\] (2)

where:
\(\alpha\) - directional cosine of the magnetization angle against the rolling direction;
\(\beta\) - directional cosine of the observation angle against the rolling direction;
\(B\) - magnetic induction amplitude;
\(\lambda_e, \lambda^e\) - material dependent elongation coefficients.

The magnetostriction mechanical stresses, in any point of the transformer core, are related to
the elongations calculated from (2) by formula (3).

\[
\sigma(\alpha, \beta, B) = 2G[\lambda_e(\alpha, \beta, B) + \lambda^e(\alpha, \beta, B)]
\] (3)

where:
\(G\) - shape deflection modulus of elasticity.
In order to calculate the magnetostriction forces acting on particular elements of the transformer core it must be divided into distinctive regions in which the magnetostriction stresses would be integrated. It is obvious and natural to divide the core into the corner regions with the overlap and the straight elements such as legs and parts of yokes (Fig. 3).

![Diagram of transformer core division](image)

Fig. 3. Division of the core into regions of stress integration, directions and senses are shown of normal and tangent stress components. Small rectangles indicate the actual integration contours.

As an example the component forces acting on the left upper corner would be computed as:

\[
F_x = -l_k \left( \int_{LCY} \sigma_y dy - \int_{LCL} \sigma_x dx \right) \\
F_y = -l_k \left( \int_{LCY} \sigma_x dx - \int_{LCL} \sigma_y dy \right)
\]

(4)

where: \( l_k \) - equivalent thickness of the core.

As the result time series, with a 5 deg step (0.2778 ms), of the instantaneous values of force components for all parts of the core were calculated. 14 harmonics were then computed for each series, with the use of Discrete Fourier Transform. The instantaneous and spectral form of the force calculations results were used further on as the input in the mechanical calculations of the transformer model forced vibrations. Examples of the force trajectory and time curve are shown in Fig. 4.

![Graphs of component forces](image)

Fig. 4. Trajectory (a) and time curve (b) of the component forces acting on the left half of the core overlay (RCCU in Fig. 3).
4. MEASUREMENTS OF OPERATIONAL DEFLECTION SHAPES

In order to visualize and animate the deflection shapes of the transformer working under normal no-load condition two-channel measurements of the vibration velocity were made. One of the accelerometers was kept in one place as the “Reference Transducer” and the second was the “Roving Transducer” measuring the phase shift and amplitude spectra in over a hundred Degrees Of Freedom (measurement points). The block diagram of the measurement system and the model transformer core with transducer mounts are shown in Fig. 5.

Fig. 5. Block diagram of the measurement system and the model transformer core with transducer mounts.

The results of the measurements were then processed to obtain deflection shapes for each harmonic, full spatial (3D) and also flat (2D) in the symmetry plane parallel to the lamination. Examples of the deflection modes for the main harmonic (100 Hz) are shown in Fig. 6.

![Deflection Shapes](image)

Fig. 6. Examples of measured deflection shapes for 100 Hz, induction 1.7 T:  a) full spatial analysis;  b) flat analysis in the symmetry plane parallel to the lamination.

The results of the measurements, shapes and maximal amplitudes of deflections, were used to verify the numerical model of the mechanical structure of the transformer and to compare with the results of forced vibrations calculations.

5. MECHANICAL CALCULATIONS OF THE TRANSFORMER MODEL

The numerical model of the mechanical structure of the transformer is used to calculate the vibrations caused by the magnetostriction forces and to identify the mechanism of vibration transmission in a 3-phase transformer. To obtain it the eigenfrequencies and modal
shapes of free vibrations must be found as well as the deflections and stresses, which would be the response to the magnetostriction forces varying in time. The structure of the transformer model was divided into several functional parts, taking into account mainly the type of material and the possibility of using adequate element types for the FEM calculations. The magnetic core was constructed from SOLID type elements in the yokes and SOLIDL elements in the legs. The SOLIDL elements imitated the lamination of the core assembled from electrotechnical sheets. The winding were built from orthotropic SHELL4L type elements. The other construction parts were simulated by SHELL4T, BEAM4T and SOLID type elements. The overall structure of the numerical model, compared to its actual laboratory model, is shown in Fig. 7.

Fig. 7. Comparison of the FEM numerical model with the actual 3-phase, 3-leg model transformer. The numerical model was verified by comparing specific mode shapes and their frequencies of free vibrations with the measurement results (Fig. 8).

Fig. 8. Comparison of the calculated modes of free vibrations with the measured resonance responses of the actual laboratory model to forced vibrations.
Static calculations of deflections caused by magnetostriction forces

Step by step, calculations were made on the mechanical model, of the static deflections caused by the instantaneous magnetostriction forces computed earlier. In order to compare the results with the measurements, the discrete time series of deflections of certain, chosen points were transformed to 14 harmonic spectra. The spectra were then processed in the same way as the measurement data. This made it possible to calculate the correlation coefficient between the mode shapes of each harmonic – measured and calculated.

Good correlation was obtained between the modes of the main harmonic 100 Hz – 0.40; and also for higher frequencies: 400 Hz – 0.52; 500 Hz – 0.29 and 600 Hz – 0.29 (correlation coefficient 1 means identical shapes).

Another way of comparing the results was to calculate the maximal elongation of the transformer legs for particular harmonics. The comparison is presented in Table 1.

Table 1. Comparison of differential elongations of transformer legs – static calculations on numerical model versus measurements on laboratory model.

<table>
<thead>
<tr>
<th>Mode frequency [Hz]</th>
<th>Model</th>
<th>Phase R</th>
<th>Phase S</th>
<th>Phase T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ampl.</td>
<td>Phase</td>
<td>Ampl.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[mm]</td>
<td>[deg]</td>
<td>[mm]</td>
</tr>
<tr>
<td>100</td>
<td>Measurement</td>
<td>238E-6</td>
<td>-14</td>
<td>382E-6</td>
</tr>
<tr>
<td></td>
<td>Calculation</td>
<td>208E-6</td>
<td>118</td>
<td>238E-6</td>
</tr>
<tr>
<td>200</td>
<td>Measurement</td>
<td>68.8E-6</td>
<td>-29</td>
<td>44.4E-6</td>
</tr>
<tr>
<td></td>
<td>Calculation</td>
<td>83.8E-6</td>
<td>169</td>
<td>28.0E-6</td>
</tr>
<tr>
<td>300</td>
<td>Measurement</td>
<td>16.0E-6</td>
<td>40</td>
<td>22.6E-6</td>
</tr>
<tr>
<td></td>
<td>Calculation</td>
<td>33.2E-6</td>
<td>-100</td>
<td>12.3E-6</td>
</tr>
<tr>
<td>400</td>
<td>Measurement</td>
<td>10.4E-6</td>
<td>-59</td>
<td>7.24E-6</td>
</tr>
<tr>
<td></td>
<td>Calculation</td>
<td>16.9E-6</td>
<td>27</td>
<td>11.7E-6</td>
</tr>
</tbody>
</table>

The best compatibility was obtained for the main harmonic – 100 Hz, which is dominant in the magnetostriction vibrations spectrum.

Dynamic calculations

The dynamic mechanical calculations are made to determine the hazard of resonance amplification of deflection amplitudes under forced vibrations. The dynamic calculations do not allow imposing the complex component forces at once, therefore they can not be the way of calculating the forced vibrations. Each harmonic must be checked one at a time. The basis for the dynamic calculations are the modes of free vibrations. The dynamic calculations take into account the individual resonance amplification coefficients and the material damping. The dynamic response is calculated as the superposition of responses of all modes (modal synthesis) of free vibrations to the frequency mode (harmonic) of the forces with respect to shape correlation, amplitude and phase.

The results of calculations for a flat (2D) system of input forces did not show any hazard of resonance.

Bearing in mind that the laboratory model showed high amplitudes of spatial deflections, in the direction perpendicular to the laminations, slight changes were made in the points of application of forces in the corners, in order to simulate a propeller like twist of the core. The actual twist of the core was noticed during assembly of the model.

Calculations made for so modified model proved that in the case of input forces slightly shifted from the symmetry plane, the response contains also spatial deflection shapes apart from the free modes.
from the flat modes. In the case of input forces of the main frequency – 100 Hz a strong amplification of spatial vibrations occurred due to the proximity of the free vibration deflection mode shape – 103.9 Hz (Fig. 8a). It must be noted that although the domination of the flat system of forces the highest deflections occur in the direction perpendicular to the symmetry plane. These amplitudes are almost 9 times higher than the maximal amplitudes in the other directions. This is consistent with the results of the measurements – see Table 2.

Table 2. Comparison of deflection amplitudes in the direction perpendicular to the symmetry plane, for spatial system of input forces – 100 Hz.

<table>
<thead>
<tr>
<th>Measurement point</th>
<th>FEM model node</th>
<th>Amplitude in direction Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Measurement</td>
</tr>
<tr>
<td>UZ</td>
<td>UZ</td>
<td>744.0</td>
</tr>
<tr>
<td>4/3</td>
<td>1662</td>
<td>799.0</td>
</tr>
<tr>
<td>10/3</td>
<td>2645</td>
<td>655.0</td>
</tr>
<tr>
<td>18/3</td>
<td>2855</td>
<td>842.0</td>
</tr>
<tr>
<td>24/3</td>
<td>1872</td>
<td></td>
</tr>
<tr>
<td><strong>Mean values</strong></td>
<td></td>
<td><strong>760.0</strong></td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

1. Quasi spatial representation of the magnetic field distribution in the overlap zone gives good results of 2D magnetic field calculations.
2. The presented method of tensor description of the magnetostriction stress field enables calculation of equivalent forces being the cause of magnetostriction vibrations.
3. The orthotropic mechanical model, which takes into account the laminated structure of the core, gives results of forced vibration calculations comparable with the measurement data.
4. The presented calculation methodology makes it possible to estimate the amplitudes and modal shapes of the main harmonics of vibration spectra and enables to detect hazards of resonance amplification of vibrations, also in the case of minor technological inaccuracies of the core assembly.

The conclusions given above prove the thesis presented at the beginning.

6. REFERENCES